Topic 09 – Further Straight Line (Solutions)

Q1, (Jun 2014, Q2)

A is the point (1, 5) and B is the point (6, −1). M is the midpoint of AB. Determine whether the line with equation $y = 2x - 5$ passes through M. **[3]**

$$M = \left(\frac{1+6}{2}, \frac{5+-1}{2}\right) = \left(\frac{7}{2}, 2\right)$$

Let $x = \frac{7}{2} \Rightarrow y = 2\left(\frac{7}{2}\right) - 5 = 7 - 5 = 2$

$\therefore$ When $x = \frac{7}{2}$, $y = 2$ $\therefore$ Line passes through M

Q2, (Jun 2011, Q9)

A line L is parallel to the line $x + 2y = 6$ and passes through the point (10, 1). Find the area of the region bounded by the line L and the axes. **[5]**

$x + 2y = 6 \Rightarrow 2y = 6 - x \Rightarrow y = 3 - \frac{1}{2}x$

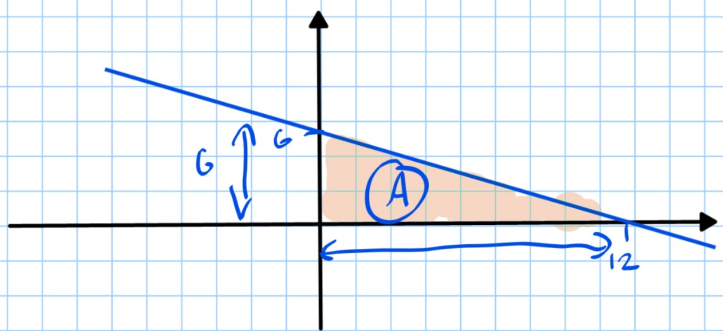
$\therefore$ Grad $= -\frac{1}{2}$ Point $= (10, 1)$

$\Rightarrow y - 1 = -\frac{1}{2}(x - 10)$

$\Rightarrow y - 1 = -\frac{1}{2}x + 5 \Rightarrow y = -\frac{1}{2}x + 6$

y-int: $(0, 6)$ x-int: $y = 0 \Rightarrow 0 = -\frac{1}{2}x + 6$

$\Rightarrow \frac{1}{2}x = 6 \Rightarrow x = 12$



$A = \frac{1}{2} \times 6 \times 12 = 36$

Q3, (Jun 2016, Q10i)

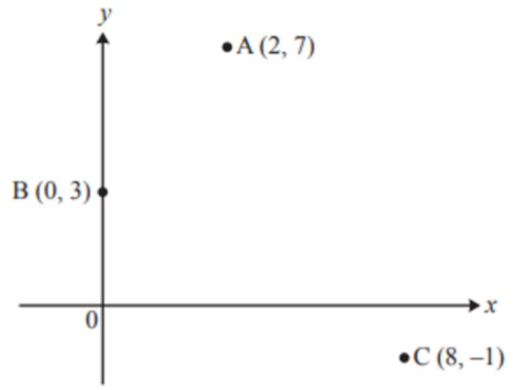
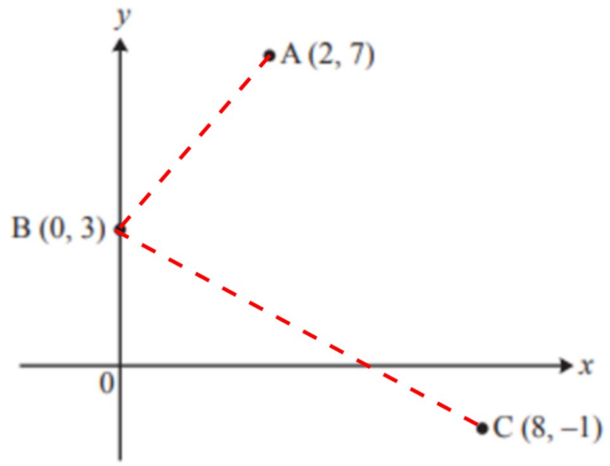


Fig. 10

Prove that angle ABC is 90°. **[3]**



$$\text{Grad}(AB) = \frac{7-3}{2-0} = 2$$

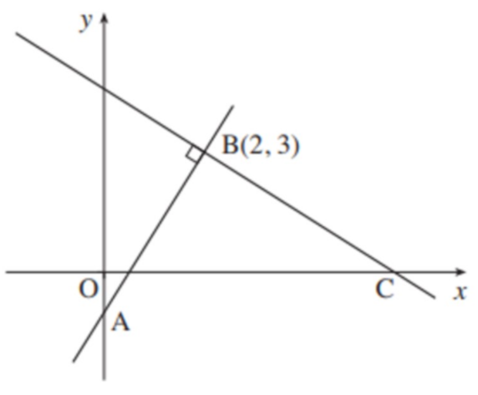
$$\text{Grad}(BC) = \frac{-1-3}{8-0} = \frac{-4}{8} = -\frac{1}{2}$$

$$2 \times -\frac{1}{2} = -1$$

$\therefore$ AB and BC are perpendicular

$\therefore \widehat{ABC} = 90°$

Q4, (Jan 2006, Q7)



Not to scale

Fig. 7

The line AB has equation $y = 4x - 5$ and passes through the point B(2, 3), as shown in Fig. 7. The line BC is perpendicular to AB and cuts the x-axis at C. Find the equation of the line BC and the x-coordinate of C. [5]

Grad of AB $= 4 \Rightarrow$ Grad of BC $= -\frac{1}{4}$, Point $= (2, 3)$

$\Rightarrow y - 3 = -\frac{1}{4}(x - 2) \Rightarrow 4y - 12 = -x + 2 \Rightarrow \boxed{x + 4y - 14 = 0}$

Let $y = 0 \Rightarrow x - 14 = 0 \Rightarrow$ $\boxed{x \text{ coord of } C \text{ is } 14}$

Q5, (Jan 2007, Q12)

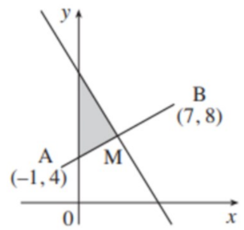
Use coordinate geometry to answer this question. Answers obtained from accurate drawing will receive no marks.

A and B are points with coordinates $(-1, 4)$ and $(7, 8)$ respectively.

(i) Find the coordinates of the midpoint, M, of AB.

Show also that the equation of the perpendicular bisector of AB is $y + 2x = 12$. [6]

(ii) Find the area of the triangle bounded by the perpendicular bisector, the y-axis and the line AM, as sketched in Fig. 12. [6]



Not to scale

Fig. 12

i/ $M = \left(\frac{-1+7}{2}, \frac{4+8}{2}\right)$

$= \boxed{(3, 6)}$

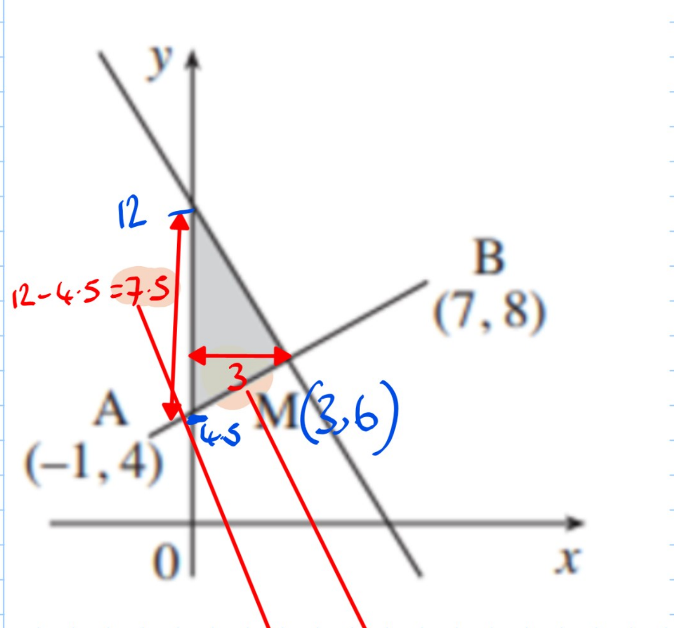
Grad of AB $= \frac{8-4}{7--1} = \frac{4}{8}$

$= \frac{1}{2}$

$\therefore$ Grad of perp bisector $= -2$

$\therefore\ y - 6 = -2(x-3) \Rightarrow y - 6 = -2x + 6 \Rightarrow \boxed{y + 2x = 12}$

ii/



For perp bisector:

Let $x = 0 \Rightarrow y = 12$

$\therefore$ y-intercept $= 12$

For line AB:

Point $= (7, 8)$, Grad $= \frac{1}{2}$

$\Rightarrow y - 8 = \frac{1}{2}(x - 7)$

$\Rightarrow y - 8 = \frac{1}{2}x - \frac{7}{2}$

$\Rightarrow y = \frac{1}{2}x + \frac{9}{2}$

$\therefore$ y-int $= \frac{9}{2}$ $(= 4.5)$

$$\boxed{\therefore \text{Area} = \frac{1}{2}(7.5)(3) = \frac{45}{4}}$$

Q6, (Jan 2013, Q11)

(i) Points A and B have coordinates (–2, 1) and (3, 4) respectively. Find the equation of the perpendicular bisector of AB and show that it may be written as $5x + 3y = 10$. **[6]**

(ii) Points C and D have coordinates (–5, 4) and (3, 6) respectively. The line through C and D has equation $4y = x + 21$. The point E is the intersection of CD and the perpendicular bisector of AB. Find the coordinates of point E. **[3]**

i/ Grad of AB $= \frac{4-1}{3--2} = \frac{3}{5}$ $\Rightarrow$ Grad of perp $= -\frac{5}{3}$

Midpoint $= \left(\frac{-2+3}{2} + \frac{1+4}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right)$

$\Rightarrow y - \frac{5}{2} = -\frac{5}{3}\left(x - \frac{1}{2}\right)$

$\Rightarrow 3y - \frac{15}{2} = -5x + \frac{5}{2}$ $\Rightarrow 5x + 3y = \frac{15}{2} + \frac{5}{2}$

$\Rightarrow \boxed{5x + 3y = 10}$

ii/ Bisector of AB: $5x + 3y = 10$

Line CD: $4y = x + 21 \Rightarrow x = 4y - 21$

$\Rightarrow 5(4y - 21) + 3y = 10$

$\Rightarrow 20y - 105 + 3y = 10$

$\Rightarrow 23y = 115 \Rightarrow y = 5$ $\Rightarrow x = 4(5) - 21 = -1$

$\therefore E(-1, 5)$

Q7, (Jun 2012, Q10)

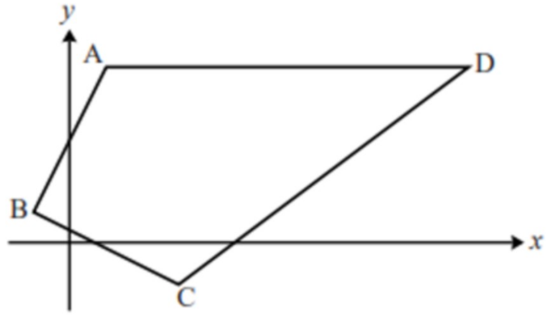


Fig. 10

Fig. 10 is a sketch of quadrilateral ABCD with vertices A (1, 5), B (–1, 1), C (3, –1) and D (11, 5).

(i) Show that AB = BC. **[3]**

(ii) Show that the diagonals AC and BD are perpendicular. **[3]**

(iii) Find the midpoint of AC. Show that BD bisects AC but AC does not bisect BD. **[5]**

i/ $|AB| = \sqrt{(-1-1)^2 + (1-5)^2} = 2\sqrt{5}$

$|BC| = \sqrt{(3--1)^2 + (-1-1)^2} = 2\sqrt{5}$

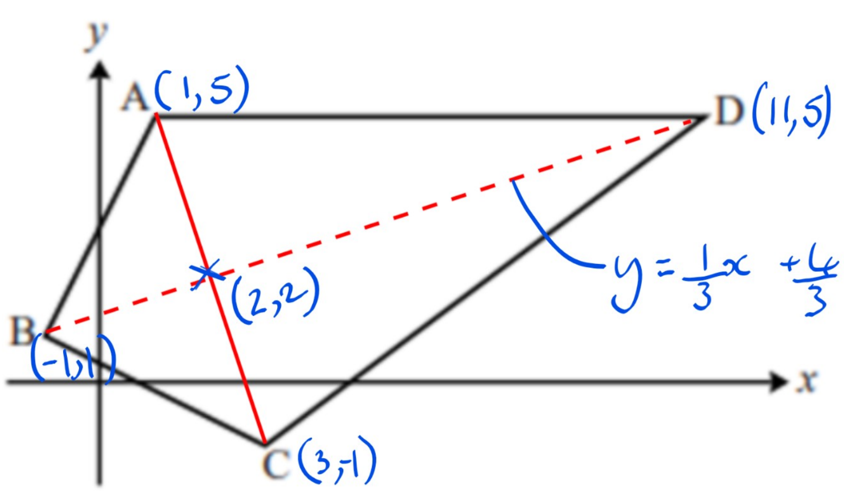
$\therefore AB = BC$

ii/ Grad of AC $= \dfrac{-1-5}{3-1} = \dfrac{-6}{2} = -3$

Grad of BD $= \dfrac{5-1}{11--1} = \dfrac{4}{12} = \dfrac{1}{3}$

$\dfrac{1}{3} \times -3 = -1 \quad \therefore$ AC and BD are perpendicular

iii/ MP of AC $= \left(\dfrac{1+3}{2}, \dfrac{5-1}{2}\right) = \boxed{(2,2)}$



Grad of BD $= \dfrac{1}{3}$

$\Rightarrow y - 1 = \dfrac{1}{3}(x - -1)$

$\Rightarrow y - 1 = \dfrac{1}{3}x + \dfrac{1}{3}$

$\Rightarrow y = \dfrac{1}{3}x + \dfrac{4}{3}$

Checking if (2,2) is on BD: Let $x = 2 \Rightarrow y = \dfrac{1}{3}(2) + \dfrac{4}{3} = \dfrac{6}{3} = 2$

$\therefore$ BD bisects AC

Midpoint of BD $= \left(\frac{-1+11}{2}, \frac{1+5}{2}\right) = (5,3) \neq (2,2)$

$\therefore$ AC does not bisect BD